B. Sc. Semester III (Honours) Examination, 2020 (CBCS)

Subject: Physics

Paper: CC-V

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Answer any eight of the following questions (All questions carry equal marks): 5×8=40

1. Define Error function and Gamma function.

   Prove that $\int_0^{\infty} e^{-y^2-2ay}\, dy = \frac{\sqrt{\pi}}{2} e^{a^2}[1 - \text{erf}(a)]$ 5

2. Evaluate $\Gamma(3.5)$

   Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\, d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$. 5

3. Expand $f(x) = \begin{cases} -x+1 & -\pi \le x \le 0 \\ x+1 & 0 \le x \le \pi \end{cases}$ in Fourier series and hence deduce the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ 5

4. State Parseval's theorem.

   If $f(x) = x \text{ in } 0 \text{ to } l$, $prove\ that\ \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} \ldots . = \frac{\pi^4}{96}$ 5

5. Solve the following equation in power series;

   $2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1 - x^2)y = 0$ 5

6. Write down Bessel's differential equation. What types of physical problems give rise to such equation?

   Prove that $J_n(x+y) = \sum_{k=-\infty}^{+\infty} J_k(x) J_{n-k}(y)$ 5

7. Using recurrence relations for Legendre Polynomial,

   Prove that $\int_{-1}^{+1} (x^2-1) P_{n+1} P_n'\, dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$. 5

8. What is the method of least square of curve fitting? Use this method to fit a straight line to the four points (1, 1.7); (2, 1.8); (3, 2.3); (4, 3.2) in the xy plane. 5

9. A semi infinite solid with diffusivity h^2 fills the positive side of yz plane. If the temperature at one face (x = 0) be given as sinusoidal function of time and it is same for all values of y and z, find the temperature throughout the solid as a function of (x,t) when the periodic state is established. 5

10. Find the displacement of a unit square membrane having unit wave velocity along it, when initial velocity is zero and initial deflection is $k \sin 2\pi x \sin \pi y$. 5