B.A/B.Sc. 1st Semester (Honours) Examination, 2021 (CBCS)
Subject: Mathematics
Course: BMH1CC01
(Calculus, Geometry and Differential Equation)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 5×6 = 30

(a) (i) Find the points of inflection on the curve $y = (log x)^3$.

(ii) Suppose you own a rare book whose value is modeled as $300e^{\sqrt{3t}}$ after t years from now. If the prevailing rate of interest remains constant at 8% compounded annually, when will be most advantageous time to sell it? [2+3]

(b) (i) Trace the curve $r = asin3\theta$.

(ii) Evaluate $\frac{d}{dx}(tanh^{-1}x)$for $x < 1$. [3+2]

(c) (i) If $I_n = \int \frac{sinn\theta}{sin\theta} d\theta$ then prove that $(n-1)(I_n - I_{n-2}) = 2\sin(n-1)\theta$.

(ii) Obtain a reduction formula for $\int sec^n \theta d\theta$ for positive integral values of $n > 1$. [2+3]

(d) Find the surface area of the solid generated by revolving the astroid $x = acos^3\theta, y = asin^3\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$ about the x-axis. [5]

(e) (i) What does the equation $11x^2 + 16xy - y^2 = 0$ become on turning the axes through an angle $tan^{-1}\left(\frac{1}{2}\right)$?

(ii) Show that the triangle formed by the pole and the points of intersection of the circle $r = 4cos\theta$ with the line $rcos\theta = 3$ is an equilateral triangle. [2+3]

(f) (i) Obtain the equation of the cylinder whose guiding curve is $2x^2 + 3y^2 - 2xy + 4x + 6y = 10, z = 0$ and whose generators are parallel to a fixed line with direction ratios l, m, n.

(ii) Show that the quadric surface given by the following equation is an ellipsoid $2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z - 5 = 0$. Find its centre. [3+2]

(g) (i) A differential equation $5\frac{dx}{dt} - x = 0$ is applicable over $|t| < 10$. If $x(4) = 10$, find $x(-5)$.

(ii) Find the integrating factor for the following differential equation: $y(1 + xy)dx - xdy = 0$. [2+3]

(h) (i) Solve: $\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{tan^{-1}x}}{1+x^2}$.

(ii) Find the equation of the curve whose slope at any point is equal to $(x + y + 1)^{-1}$ and passes through the point (1,0). [2+3]

2. Answer any three questions: 10×3 = 30

(a) (i) Find a and b such that $\lim_{x\to 0} \frac{ae^x + be^{-x} + 2\sin x}{\sin x + x\cos x} = 2$.

(ii) Find the envelope of the family of circles described on OP as diameter where O is the origin and P is a point on the curve $xy = a^2$.

(iii) Find all the asymptotes of $x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0$. [3+4+3]

(b) (i) Find the perimeter of the hypo-cycloid $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$.

(ii) Prove that the surface area of the solid generated by revolving the cycloid $x = a(\theta + sin\theta), y = a(1 + cos\theta)$ is $\frac{64}{3}\pi a^2$.

(iii) If $I_n = \int_0^1 (1 - x^2)^n \, dx$, prove that $(2n + 1)I_n = 2nI_{n-1}$. [3+4+3]

(c) (i) A variable sphere passes through $(0,0,\pm c)$ and cut the lines $y = x, z = c$ and $y = -x, z = -c$ in points, whose mutual distance is $2a$. Show that the centre of this sphere lies on the circle $z = 0,\ x^2 + y^2 = a^2 - c^2$.

(ii) Lines are drawn through the origin to meet the circle, in which the plane $x + y + z = 1$ cuts the sphere $x^2 + y^2 + z^2 - 4x - 6y - 8z + 4 = 0$. Show that they lie on the cone $x^2 - y^2 - 3z^2 - 6yz - 4zx - 2xy = 0$ and meet the sphere again at points on the plain $y + 2z - 2 = 0$.

(iii) Find the equations of the generators to the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ which pass through the point $(acos\alpha, bsin\alpha, 0)$. [3+4+3]

(d) (i) Solve: $y^2 \frac{dy}{dx} - 2\frac{y^3}{x} = 2x^2$, given that when $x = 1, y = 1$.

(ii) Solve: $y^2 logy = xyp + p^2, p \equiv \frac{dy}{dx}$.

(iii) By the substitution $x^2 = u, y - x = v$ reduce the differential equation $xp^2 - 2yp + x + 2y = 0$ to Clairaut's form and find its singular solution, if any. [3+3+4]

(e) (i) If $y = (sin^{-1}x)^2$ then show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0.$$

(ii) Determine the length of the cissoid $r = 2atan\theta sin\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{4}$

(iii) Determine the nature of the conic $x^2 + 9y^2 + 6xy + 3x + 6y - 4 = 0$. [3+4+3]