B.A 5th Semester (General) Examination, 2020 (CBCS)

Subject: Mathematics

Course: BAMATH5GE1 (Calculus)

Time: 3 Hours   Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) State and prove Leibnitz's theorem on successive differentiations. [5]

(b) If $f(x) = ax^2+2hxy+by^2$ find f_x and f_y. [5]

(c) If $u = sin^{-1}\frac{x}{y} + tan^{-1}\frac{y}{x}$ then show that $x\frac{\partial u}{dy} + y\frac{\partial u}{\partial y} = 0$. [5]

(d) Show that $\int_o^1 \frac{log(1+x)}{1+x^2}dx = \frac{\pi}{8}log2$. [5]

(e) Evaluate the integral: $\int_0^{\frac{\pi}{2}} sin^n\, x\, dx$. [5]

(f) Solve: $(x^3+y^3)dx-xy^2dy=0$. [5]

(g) Solve: $\frac{dy}{dx} + x\,sin\,2y = x^3\ cos^3\,y$. [5]

(h) Show that $cos\,y\,dx + (1+e^x)\,sin\,y\,dy = 0$. [5]

2. Answer any three questions: 10×3 = 30

(a) (i) Prove that the area of whole ellipse is πab . [5]

(ii) Solve: $y(1+xy)dx - xdy = 0$. [5]

(b) (i) Find the radius of curvature at the point (r,θ) on the cardioide, $r = \alpha(1-cos\,\theta)$. [5]

(ii) Find the condition that the conics $ax^2+by^2=1$ and $cx^2+dy^2=1$ will cut orthogonally. [5]

(c) (i) Find the derivative of the function $f(x)=x^3+2x$ from the first principles. [5]

(ii) Find the asymptotes of the curve $2x(y-5)^2=3(y-2)(x-1)^2$. [5]

(d) (i) Use reduction formula to evaluate $\int_0^{\frac{\pi}{2}} cos^9\, x\mathrm{d}x$. [5]

(ii) Show that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. [5]

(e) (i) Solve: $(x^2+y^2)dx-2xydy=0$. [4]

(ii) Show that $lim_{x\to 0}\frac{(1-x)^n-1}{x} = 0$. [6]