B.A/B.Sc 5th Semester (General) Examination, 2020 (CBCS)

Subject: Mathematics

Course: BMG5DSE1A1 (Matrices)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: $6\times5 = 30$

(a) If μ is an eigen value of a non-singular matrix A, then prove that $1/\mu$ is an eigen value of A^{-1}. [5]

(b) Find the dimension of the subspace S of $\mathbb{R}^3$ defined by $S=\{(x, y, z): x+2y=z, 2x+3z=y\}$. [5]

(c) (i) Determine k so that the set $S=\{(1, 2, 1),(k, 3, 1), (2, k, 0)\}$ is linearly dependent in $\mathbb{R}^3$. [3]

(ii) Examine if the set $S=\{(x, y, z): x+y+z=1\}$ is a subspace of $\mathbb{R}^3$. [2]

(d) If μ is an r-fold eigen value of an $n\times n$ matrix A , then prove that rank of $(A-\mu I_n) \geq n-r$. [5]

(e) (i) Show that the sum of the roots of the characteristic equation of a square matrix A is the trace of A. [3]

(ii) If A and B are two matrices such that $AB=0$ where 0 is the zero matrix, can you conclude that either A or B is zero matrix? Justify your answer. [2]

(f) If $A=\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$ then show that $A^2-4A+3I_2=0$. Hence find A^{-1} and solve the system of equations $2x-y=3$ and $-x+2y=-3$. [5]

(g) Define basis of a vector space. Let V be a vector space with $\{\alpha, \beta, \mu\}$ as basis. Prove that $\{\alpha+\beta+\mu, \beta+\mu, \mu\}$ is also basis of V. [1+4]

(h) (i) Find a basis containing the vectors $(1,0,0)$ and $(1,1,0)$ for the vector space $\mathbb{R}^3$. [3]

(ii) Define the rank of a matrix. [2]

2. Answer any three questions: $10\times3 = 30$

(a) (i) If A is a square matrix of order n, prove that $adj\,(adjA)=|A|^{n-2}A$. [5]

(ii) If $A=\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & 4 \\ 3 & 3 & 7 \end{bmatrix}$, use elementary row operation on A find A^{-1}. [5]

(b) (i) Find the fully reduced normal form of the matrix $A=\begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & 4 & 6 \\ 3 & 0 & 7 & 9 \end{bmatrix}$. [5]

(ii) Prove that the eigen values of a real symmetric matrix are all real. [5]

(c) (i) Solve the system of equations $x+2y+z=1$, $3x+y+2z=3$, $x+7y+2z=1$. [5]

(ii) Prove that a square matrix A over a field F is invertible if and only if A is non-singular. [5]

(d) (i) State and prove a necessary and sufficient condition for a square matrix A of order n over a field F to be diagonalizable. [6]

(ii) If V is a vector space over a field F, then prove that intersection of two subspaces of V is also a subspace of V. [4]

(e) (i) Let V be a vector space of dimension n over a field F. Prove that any linearly independent set of n vectors of V is a basis of V. [4]

(ii) Find a non-singular matrix P such that $P^{-1}AP$ is a diagonal matrix where [6]

$$A=\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

B.A/B.Sc 5th Semester (General) Examination, 2020 (CBCS)

Subject: Mathematics

Course: BMG5DSE1A2 (Mechanics)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: $6\times 5 = 30$

(a) (i) A particle describes a parabola $r = a\sec^2\dfrac{\theta}{2}$ such that the cross-radial component of velocity of the particle is constant. Show that its radial acceleration is a constant. [2]

(ii) A particle is placed on a rough plane, whose inclination to the horizon is α, and is acted upon by a force P acting parallel to the plane and in a direction making an angle β with the line of greatest slope in the plane; if the coefficient of friction be μ and the equilibrium be limiting, find the direction in which the body will begin to move. [3]

(b) The moments of a system of coplanar forces acting in the xy-plane about $(0,0), (a,0), (0,a)$ are $a\omega, 2a\omega, 3a\omega$ respectively. Find the components parallel to the coordinate axes and the line of action of the single force to which the system is equivalent. [5]

(c) Find the centre of gravity of a plane lamina of uniform density in the form of a quadrant of an ellipse. [5]

(d) A particle oscillating harmonically in a straight line has velocities v_1, v_2 and accelerations f_1, f_2 at two of its positions on the path . If d be the distance [5] between the two positions, show that $d=\dfrac{v_1^2-v_2^2}{f_1+f_2}$.

(e) A particle moves in the curve $y = a\, log_e sec(x/a)$ in such a way that the tangent to the curve rotates uniformly. Prove that the resultant acceleration of the particle varies as the square of the radius of curvature. [5]

(f) A particle is projected with a velocity u in a direction inclined at an angle α to the horizontal. Show that the sum of the kinetic and potential energies of the particle is constant throughout the motion. [5]

(g) Prove that the kinetic energy of two particles of masses m and $\acute{m}$ moving in a plane is $\frac{1}{2}(m+\acute{m})V^2+\frac{1}{2}\frac{m\,\acute{m}}{(m+\acute{m})}v^2$ where V is the velocity of the centre of mass of the particles and v the velocity of either of them relative to each other. [5]

(h) A square frame ABCD of four equal jointed rod is hanging from A, the shape being maintained by a string joining mid points of AB and BC. Prove that the ratio of tension of the string to the reaction at C is $\frac{8}{\sqrt{5}}$. [5]

2. Answer any three questions: 10×3 = 30

(a) (i) A solid hemisphere is supported by a string fixed to a point on its rim and to a point on smooth wall with which the curved surface of the hemisphere is in contact. If θ, φ are the inclination of the string and the plane base of the hemisphere to the vertical, prove that $tan\varphi = \frac{3}{8} + tan\theta$. [5]

(ii) An isosceles triangular lamina is such that its mass per unit area at every point is proportional to the sum of the distances of the part from the equal sides of the triangle. Prove that the distance of centre of gravity from vertex is $(3/4)^{th}$ of the altitude. [5]

(b) (i) A rifle bullet losses $(1/16)^{th}$ of its velocity in passing through a wooden board. Assuming that the resistance of the boards to be uniform, find how many such uniform boards it would pass through before being stopped. [4]

(ii) Deduce the expressions for the tangential and normal components of velocities and accelerations of a particle moving on a plane curve. [6]

(c) (i) A particle is projected vertically upwards under gravity with a velocity U in a medium of resistance kv per unit mass, where v is the velocity of the particle at any instant and k is a constant. Show that the greatest height attained by the particle is $\frac{U}{k}-\frac{g}{k^2}log\left(1+\frac{kU}{g}\right)$. [5]

(ii) A particle P possesses two constant velocities u and $2u$ such that u is always parallel to a fixed direction OX and $2u$ is always perpendicular to the radius vector OP. Show that the path describes is an ellipse with focus at O and eccentricity $\frac{1}{2}$. [5]

(d) (i) A particle is performing an S.H.M. of period T about a centre O and it passes through a point P with a velocity v in the direction OP. If the particle returns to P, in time t, then show that the distance OP is $\frac{2\pi}{vT}\cot\frac{\pi t}{T}$. [5]

(ii) The height of a solid homogeneous right circular cone is h and the radius of its base is [5]

r; a string is fastened to its vertex and to a point, on the circumference of circular base, and is then put over a small smooth peg. If the cone rests in equilibrium with its axis horizontal, prove that the length of the string is $\sqrt{h^2 + 4r^2}$.

(e) (i) Find the centre of gravity of the area enclosed by the curves $y^2 = ax$ and $x^2 = by$. [5]

(ii) Obtain the equation of motion of a plane lamina rotating about a fixed axis perpendicular to the plane of the lamina in the form $MK^2\ddot{\theta}$=L, the symbols are to be explained by you. [5]

B.A/B.Sc 5th Semester (General) Examination, 2020 (CBCS)

Subject: Mathematics

Course: BMG5DSE1A3 (Linear Algebra)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) (i) Define vector space and give an example of it. [2]

(ii) Prove that the intersection of two subspaces of a vector space V over a field F is a subspace of V. [3]

(b) Let V be a vector space over the filed F and α, β, γ be in V. If $S = \{\alpha, \beta, \gamma\}$ and $T = \{\alpha + \beta, \alpha, \alpha + \beta + \gamma\}$ then prove that $L(S) = L(T)$. [5]

(c) (i) For what value of k does the set of vectors $\{(k, 1, 1), (1, k, 1), (1, 1, k)\}$ form a basis of $\mathbb{R}^3$. [2]

(ii) Find the dimension of the subspace S of $\mathbb{R}^3$ defined by [3]

$S = \left\{(x, y, z) \in \mathbb{R}^3 : 2x + y - z = 0\right\}$.

(d) Two subspaces of $\mathbb{R}^3$ are $U = \{(x, y, z) : x + y + z = 0\}$ and $W = \{(x, y, z) : x + 2y - z = 0\}$. Find $\dim U$, $\dim W$, $\dim U \cap W$. [5]

(e) Let V and W be vector spaces over F. Let $T: V \to W$ be a linear mapping. Then prove that T is injective if and only if $Ker\ T = \{\theta\}$. [5]

(f) Find a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ such that $Ker\ T$ is the subspace $U = \{(x, y, z) : x - y - z = 0\}$ of $\mathbb{R}^3$. [5]

(g) Show that the set of all matrices of order 2×2 over real numbers is a vector space over the field of real numbers. [5]

(h) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$. [1+4]

2. **Answer any three questions:** 10×3 = 30

(a) (i) Let V and W be vector spaces over F and $T: V \to W$ be a linear mapping. Prove that $Ker\ T$ is a subspace of V. [5]

(ii) Find the dimension of $Ker\ T$ where $T:\mathbb{R}^3 \to \mathbb{R}^3$ is defined by [5]

$$T(x, y, z) = (x + 2y + 3z, 3x + 2y + z, x + y + z).$$

(b) (i) Let $\dim V = \dim W = n$ and $T: V \to W$ be a linear mapping. Prove that T is one-one if and only if T is onto. [5]

(ii) Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation given by $T(x, y, z) = (x + 2y + 3z, x + 3y + 2z)$. Find the dimension of $Ker\ T$ and ImT [5]

(c) (i) When is a linear transformation called an isomorphism? Give an example of an isomorphism. [4]

(ii) Let $T:\mathbb{C} \to \mathbb{R}^2$ be a mapping given by $T(a + ib) = (a, b)$ for all $a + ib \in \mathbb{C}$. Examine whether T is an isomorphism, $\mathbb{C}$ being the vector space of all complex numbers over the field $\mathbb{R}$. [6]

(d) (i) Prove that any linear transformation on a finite dimensional vector space onto itself is an isomorphism. [5]

(ii) Let $T:\mathbb{R}^2 \to \mathbb{R}$ where $T(1,1) = 3$ and $T(0,1) = -2$. Find $T(x, y)$. [5]

(e) (i) Let the linear mapping $T:\mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (2x + y - 3z, y + 4z, x - y + 3z)$. Find the matrix T relative to the ordered basis $\{(0,1,1), (1,0,1), (1,1,0)\}$ of $\mathbb{R}^3$. [5]

(ii) Let the linear mapping $T:\mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (2x, 4x - y, 2\,x + 3y + z)$. Show that T is invertible and find the inverse of T. [5]