B.A/B.Sc. 1st Semester (General) Examination, 2021 (CBCS)
Subject: Mathematics
Paper: BMG1CC1A/MATH-GE1 (Differential Calculus)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) (i) Show that $\lim_{x\to 0} x\sin(1/x)=0$. [2]

(ii) The function f is defined as follows: [3]

$$f(x)=\begin{cases} -2\sin x & if\ -\pi\le x\le -\frac{\pi}{2} \\ a\sin x+b, & if\ -\frac{\pi}{2}<x<\frac{\pi}{2} \\ \cos x & if\ \frac{\pi}{2}\le x\le \pi \end{cases}$$

If $f(x)$ is continuous in the interval $-\pi\le x\le\pi$, find the values of the constants a and b.

(b) (i) Give the geometrical interpretation of Rolle's theorem. [2]

(ii) Determine all the numbers in [-1, 2] for which the conclusions of the Mean Value Theorem for the following function is satisfied. [3]

$f(x)=x^3+2x^2-x,\ x\in[-1,2]$.

(c) State and prove Euler's theorem on homogeneous function in case of two variables. [2+3]

(d) (i) Where does the function $f(x)=\sin 3x-3\sin x$ attain its maximum or minimum value in $(0,2\pi)$? [3]

(ii) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x}$. [2]

(e) Show that $\frac{x}{1+x}<\log(1+x)<x$, if $x>0$. [5]

(f) (i) If $y=x^{2n}$, where n is a positive integer, show that [3]

$y_n=2^n\{1.3.5\ldots.(2n-1)\}x^n$.

(ii) If $y=A\sin mx+B\cos mx$, prove that $y_2+m^2y=0$. [2]

(g) Show that the tangent at (a,b) to the curve [5]

$\left(\frac{x}{a}\right)^3+\left(\frac{y}{b}\right)^3=2$ is $\frac{x}{a}+\frac{y}{b}=2$.

(h) Find the envelope of the straight lines $y=mx+\sqrt{a^2m^2+b^2}$. [5]

2. Answer any three questions: 10×3 = 30

(a) (i) If $y = e^{a\sin^{-1}x}$, then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$. [5]

(ii) If $V = z\tan^{-1}\frac{y}{x}$, then prove that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$. [5]

(b) (i) If $lx + my = 1$ is a normal to the parabola $y^2 = 4ax$, then show that $al^3 + 2alm^2 = m^2$. [5]

(ii) If ρ_1 and ρ_2 be the radii of curvature at the ends of the focal chord of the parabola $y^2 = 4ax$, then show that $\rho_1^{-\frac{2}{3}} + \rho_2^{-\frac{2}{3}} = (2a)^{-\frac{2}{3}}$. [5]

(c) (i) Is the origin a double point on the curve $y^2 = 2x^2y + x^4y - 2x^4$? If so, state its nature. [2+3]

(ii) Trace the curve $r = a\sin 2\theta$. [5]

(d) (i) Given $xy = 4$, find the maximum and minimum values of $4x + 9y$. [5]

(ii) Find the asymptotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$. [5]

(e) (i) State Lagrange's mean value theorem and examine whether it is applicable to the function $f(x) = 4 - (6-x)^{\frac{2}{3}}$ in the interval $[5,7]$? [2+3]

(ii) Expand $\log_e(1+x)$ in a finite series in powers of x, with the remainder in Lagrange's form. [5]