B.A/B.Sc 3rd Semester (General) Examination, 2020 (CBCS)
Subject: Mathematics (General/Generic)
Course: BMG3CC1C/MATH-GE3 (Real Analysis)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. **Answer any six questions:** 6×5 = 30

(a) (i) If $y > 0$, then show that there exists a natural number n such that $\frac{1}{2^n} < y$. [2]

(ii) State the least upper bound axiom for the set $\mathbb{R}$ of real numbers, and hence deduce the Archimedean property of $\mathbb{R}$. [3]

(b) (i) When is a subset *I* of the set of real numbers said to be an interval? [2]

(ii) Is the intersection of two open intervals an open interval? Justify your answer. [3]

(c) (i) Show that the sequence $\{\frac{(-1)^n}{n}\}$ is a null sequence. [2]

(ii) State Bolzano-Weierstrass theorem and verify it for the set $S=\{\frac{n}{n+1}, n = 1, 2, 3, \ldots\}$. [3]

(d) Give an example of a sequence of irrational numbers which converges to a rational number. [5]

(e) (i) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n} \sin\frac{1}{n}$ is convergent. [2]

(ii) If $\sum_{n=1}^{\infty} a_n$ isa convergent series of positive and non-increasing terms, then prove that $\{a_n\}$ is a null sequence. [3]

(f) (i) When is a series said to be an alternating series? [1]

(ii) Show that the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sqrt{n}}$ is convergent. Examine its absolute convergence. [2+2]

(g) (i) Show that the sequence of functions $\{f_n\}$ defined by $f_n(x) = \frac{x}{1+nx}$, where *x* is non negative real, is pointwise convergent. [2]

(ii) Let *A* be a subset of the set of real numbers, and the sequence of functions $\{f_n\}$ defined on *A* converges pointwise to a function *f* on *A*. If $M_n = \sup_x |f_n(x) - f(x)|$, *x* runs over *A*. Show that the sequence $\{f_n\}$ is uniformly convergent on *A* to *f* if and only if $\{M_n\}$ is a null sequence. [3]

(h) Let a function f be defined by $f(x) = \sum_{n=1}^{\infty} \frac{\cos nx}{10^n}$, *x* being any real. Show that *f* is continuous. [5]

2. **Answer any three questions:** 3×10 = 30

(a) (i) Define a rational number. Show that the set *Q* of all rational numbers is countable. [1+4]

	(ii)	Show that the sequence $\{\left(1+\frac{1}{n}\right)^n\}$ is convergent .	[5]
(b)	(i)	Let $\{u_n\}$ be a null sequence and $\{v_n\}$ be a bounded sequence. Is $\{u_n v_n\}$ a null sequence? Justify your answer.	[5]
	(ii)	Show that a sequence $\{u_n\}$ is convergent if and only if for each $\mu > 0$, there exists a natural number k such that $\lvert u_{n+p} - u_n\rvert < \mu$ for all n greater than or equal to k, and p=1, 2, 3,…	[5]
(c)	(i)	State and prove Cauchy's first theorem on limits.	[5]
	(ii)	Apply Cauchy's general principle of convergence to prove that the sequence $\{1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}\}$ is not convergent.	[5]
(d)	(i)	If $\sum_{n=1}^{\infty} u_n$ is a convergent series of positive terms, examine if the series $\sum_{n=1}^{\infty} u_n^2$ is convergent.	[5]
	(ii)	Show that the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{3n-2}{n^5}$ is absolutely convergent.	[5]
(e)	(i)	Find the radius of convergence of $\Sigma\, x^n/n^2$.	[4]
	(ii)	Find the radius of convergence of the series $\sum_{n=1}^{\infty}(-1)^{\mathrm{n}}\frac{n!x^n}{\mathrm{n}^{\mathrm{n}}}$.	[2]
	(iii)	Show that the sequence of functions $\{f_n\}$ defined by $f_n(x)=\frac{nx}{1+n^2x^2}$ is not uniformly convergent in any interval containing zero.	[4]